On the Top-k Shortest Paths with Dissimilarity Constraints

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Introduction The classical k shortest paths problem (kSP) returns k shortest paths between a pair of source and destination nodes in a graph. This problem has numerous applications in various kinds of networks (road and transportation networks, communications networks, social networks, etc.) and is also used as a building block for solving optimization problems.

Let $D = (V, A)$ be a digraph with n vertices and m arcs, We define an s, t directed path as a sequence of vertices $s = v_1, v_2, \cdots, v_l = t$ s.t. $v_i \in V$ and $(v_i, v_{i+1}) \in A$ for all $1 \leq i < l$. A path is called simple if $v_i \neq v_j$ for all $0 \leq i < j \leq l$. Several algorithms for solving the kSP have been proposed. In particular, Eppstein [4] proposed an exact algorithm that computes k shortest paths (not necessarily simples) with time complexity in $O(m+n+k)$. An important variant of this problem is the k shortest *simple* paths problem $kSSP$ which adds the constraint that each path of the output must be simple. The algorithm with the best known time complexity for solving this problem has been proposed by Yen [6], with time complexity in $O(kn(m+n \log n))$. Since, several improvements have been proposed to improve the efficiency of the algorithm in practice [5].

However, the k shortest simple paths are often quite "similar", roughly it means that there is a large number of common edges among the k shortest simple paths. This is undesirable in many applications as it adversely reduces the flexibility, in particular in road networks in which diversity is a desirable feature [2].

The problem of finding dissimilar paths, where the output paths are not supposed to share a large number of common edges, have been considered by several studies [2]. Note that the output paths are not required to be vertex/edge disjoint.

Our contributions. Let $D = (V, A)$ be a directed graph (digraph for short) with vertex set V and arc set A. Let $w : A \to \mathbb{R}^+$ be the weight function of the arcs. For any path P in D, let $w(P) = \sum_{e \in A(P)} w(e)$. We define and study the following measures of similarity between paths.

Definition 1 *Let P, P' be two paths of D and let $X = \sum_{e \in A(P) \cap A(P')} w(e)$.*

- *The Jaccard similarity between P and P' is $S^J(P, P') = \frac{X}{w(P \cup P')}$;*
- *The Asymmetrical similarity of P' w.r.t. P is $S^{Asy}(P, P') = \frac{X}{w(P)}$;*
- *The Min-similarity between P and P' is $S^{Min}(P, P') = \frac{X}{\min\{w(P), w(P')\}}$;*
- *The Max-similarity between P and P' is $S^{Max}(P, P') = \frac{X}{\max\{w(P), w(P')\}}$.*

Given one of the similarity measures $Z \in \{S^J, S^{Asy}, S^{Min}, S^{Max}\}$ above and $0 \leq \theta \leq 1$, two paths P and P' are said *θ-dissimilar* (or P' is said *θ-dissimilar to* P in the case of asymmetrical similarity) for measure Z if $Z(P, P') \leq \theta$.

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In this work, we aim at studying the following general problem. Given a digraph $D = (V, A)$ with weight function $w : A \rightarrow \mathbb{R}^+$, a pair of vertices $(s, t) \in V \times V$, $k \in \mathbb{N}$, $0 \leq \theta \leq 1$ and a similarity measures $Z \in \{S^J, S^{Asy}, S^{Min}, S^{Max}\}$, the goal is to decide if there exists k θ-dissimilar paths from s to t with respect to Z. We studied the complexity of several variants of the problem of finding dissimilar paths. The results related to the complexity of the problem are summarized in Table 1.

Problem	Objective	Complexity
DISSIMILAR PATH PROBLEM (DISP*Z)	Finding an s, t path dissimilar to another given s, t path	Polynomial for $Z = S^{Asy}$
FINDING k SHORTEST DISSIMILAR s, t Path (FINDING k-SDISSZ)	Finding k s, t shortest paths dissimilar to each other	NP-COMPLETE for $k = 2$ and for all $Z \in \{S^J, S^{Asy}, S^{Min}, S^{Max}\}$
k DISSIMILAR PATH (k-DISSZ)	Finding an s, t path dissimilar to k given s, t paths	NP-COMPLETE for $k = 2$ for all $Z \in \{S^J, S^{Asy}, S^{Min}, S^{Max}\}$, and weakly NP-COMPLETE if $Z = S^{Asy}$ and the weight of the arcs are integers
SHORTEST DISSIMILAR PATH (k-SDISPZ)	Finding a shortest s, t path dissimilar to k given s, t paths	NP-COMPLETE for $k = 1$ for all $Z \in \{S^J, S^{Asy}, S^{Min}, S^{Max}\}$, and weakly NP-COMPLETE if $Z = S^{Asy}$ and the weight of the arcs are integers

TAB. 1 – The complexity of different variants of the problem of finding dissimilar paths

We also propose exact methods starting by an Integer Linear Program solving the problem, then we describe how we can adapt Yen's [6] algorithm to extract only dissimilar paths and finally we propose a pruning technique to speed up the described adaptation. Our ongoing work concerns the implementation of the over mentioned methods and the design of fast heuristics in order to compare our result with other exact methods and heuristics proposed in [1, 3].

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