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Wavelet Co-movement Estimation and Neural Network Forecasting for Energy Commodities, US Stock and US Dollar Indexes

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Abstract. The use of wavelet techniques for studying the dynamics of the time series of oil and gas prices, the Dow Jones index and the US dollar index allowed to establish some correlation relationships between volatility in the relevant markets. By means of discrete wavelet transform and continuous wavelet transform, the wavelet power spectrum of each series was constructed, wavelet coherence for time series paires was investigated, and wavelet multiple correlation was determined. In order to study the co-movement of the time series we madure wavelet multiple correlation and cross correlation of the research indicators. Common revenue movements of the studied time series characterize the behavior of the relevant markets. The levels of high volatility at similar intervals explain that there is a link between the changes in these markets, and the global economy is vulnerable to oil and gas prices, the value of the dollar index and the Dow Jones index. At the next stage, a comparison of the predictive capabilities of various neural networks is made. For series-leaders, forecasting models based on neural network of Long Short Term Memory and Wavelet based Back Propagation were built. Comparison of the forcasting errors suggests that the application of both methods on short horizons gives good modeling results.

Keywords: wavelet analysis, wavelet coherence, wavelet multiple correlation and cross correlation, neural networks, volatility.

1 Introduction

The global financial system combines various assets traded in markets. These markets have characteristics that lead to different types of volatility. Asset prices react to each other in many respects. Market participants operate at different time scales, depending on their requirements, and, therefore, the true dynamic structure of the relationship between variables can vary at different time scales. Looking at this phenomenon in terms of portfolio diversification, one can say that market participants with short-term investment horizons are active at higher frequencies, and those with long-term investment horizons operate on a longer scale. Therefore, it is necessary to analyze co-movements in the markets on several scales. Wavelet methods provide a large-scale data analysis naturally [1].

The growing interest in wavelet analysis among economic researchers and its applicability in such areas as time decomposition, forecasting and density estimation led to the emergence of various wavelet techniques for analyzing nonstationary financial time series [2]. The wavelet approach is ideally suited for studying high-frequency data generated by financial markets, providing valuable information for decision-making, as an analyst can focus on a certain amount of time when trade patterns are considered important. Thus, wavelet technique has enormous potential in economics and finances, since the relationships between different variables can be analyzed in time-frequency space. It allows to research the interconnections between variables at different frequencies and the corresponding information on the evolution of a variable in time simultaneously.

Market participants operate at different time scales depending on their requirements and, therefore, the true dynamic structure of the relationship between the variables may vary at different time scales. Considering this phenomenon in terms of portfolio diversification, it means that market participants with short-term investment horizons are active at higher frequencies, and those with longer-term investment horizons operate on longer scales. Therefore, it is necessary to analyze the joint movements in the stock markets on several scales. Wavelet methods naturally provide for large-scale data analysis [1].

Continuous wavelet transform is a promising method for analyzing the joint movement of stock prices in different countries, since this technique can illustrate the value of the share price ratio between two different markets in time-frequency space. It follows that the trend in the stock returns co-movement can be divided into short, medium and long-term horizons, which serve as an important benchmark for investors to make investment decisions in the short, medium and long term, respectively.

The purpose of the paper is to study the dynamics of oil and gas prices, Dow Jones and US dollar indexes, and to identify co-movements in relevant markets in time and frequency domains. Using wavelet methodologies, pair coherence and multiple correlation of time series returns were studied in order to determine co- movements leaders at the appropriate frequency and time scales. For these leaders, the prognostic capabilities of the Long Short Term Memory (LSTM) and Wavelet based Back Propagation (WBP) neural networks were compared.

2 Analysis of Recent Research

Economists have long been concerned about the joint movement of asset prices across markets, as it provides information that helps investors make investment decisions such as asset allocation, portfolio diversification and risk management. Over the past decades, many studies have examined the interconnection between different economic variables in different markets. Rua and Nunes [3] suggested using a continuous wavelet analysis to evaluate the co-movement of stock prices on international stock markets. Following the methodology of Rua and Nunes, the co- movement of various economic variables on different stock markets has been studied in many studies [4-10]. Distribution of profits in various energy markets was considered in [11]; the relationship between oil prices and the exchange rate was studied in [12]; the ratio between the price of oil and the price of shares was investigated in [13] as well as the correlation between different macroeconomic variables [14, 15].

It is worth noting that there are also many works that use discrete wavelet analysis to detect the interconnections between different economic variables in different countries. The discrete wavelet analysis was first proposed by Ramsey and Lampast [16, 17] to study the relationship between income and other macroeconomic variables. This technique has become very popular in applied economics since Gencay, Selcuk, Whitcher [18] and Pervical, Walden [19] presented details of the discrete wavelet method for analyzing time series [20, 21]. According to this methodology, the relationship between different economic variables, such as the co-movement of profits in different stock markets [22-24], the co-movement of long-term interest rates between European countries [25] was investigated. The global relationship between the Dow Jones Industrial Average and the US industrial index is analyzed by Gallegati [26] using wavelet correlation and cross-correlation methods.

In [27], using a wavelet approach, the relationship between four basic assets simultaneously (oil, gold, currency and stocks), between the four fear indices (OVX, GVZ, EVZ and VIX) and the link between all assets for detection of co-movement in the world financial markets. In [28] states that oil is now the most important source of energy. Any sharp drop in its prices will have beneficial effects on the US dollar and mainly for the economic competitiveness of countries that are not large oil producers, and vice versa.

As companies operating in oil, gold and forex markets sell their stocks on the stock market, one can expect stocks to represent the most important of these four assets. They are the key factors in asset allocation and, therefore, are most sensitive to global shocks [29-36].

All of the above studies are an example of the relationship between underlying assets and total volatility indices in the time domain. However, what promises the simultaneous region and area frequency (wavelet analysis) in this area of research, you can make the analysis of the co- movement more complex and useful to investors. It is expected that oil and US dollar prices will be more prone to external shocks due to the specific features of their markets, which are heavily dependent on policy interference through energy and monetary policy, to which extent these markets react to each other and the feedback between gold and stocks are even complex and fuzzy [37,38].

Barunik et al. [39] used wavelet coherence analysis to study the temporal dynamics of local correlations between the stock markets of Central Europe and Western Europe. The interdependence between the major European markets has been found to be significantly changing over time and scale.

The global relationship between the Dow Jones Industrial Average and the US Industrial Production Index is analyzed by Gallegati [26] using wavelet correlation and cross-correlation methods.

Unpredictable stock market factors make stock futures forecasting more complicated. Although the efforts in an effective prediction method developing have a long history, recent advances in the field of artificial intelligence and the use of artificial neural networks have increased success in a nonlinear approximation. In [40], it is suggested to use a combination of a futures forecasting model based on a stock index using neural networks of deep learning (an automatic encoder and a limited Boltzmann machine). High-frequency data are used to study the predictive performance of deep learning, compared with traditional artificial neural networks.

3 Research Methods

Wavelet technics based on discrete wavelet transform (DWT) and continuous wavelet transform (CWT) are used to study interconnections and interactions between time series. CWT is employed to determine the wavelet power spectrum of a signal and wavelet coherence of two signals. DWT is used to compute the multiple wavelet correlation and multiple cross-wavelet correlation of time series.

The wavelet function $\psi(t)$ is a local function, both in time and in frequency, and it is defined as:

$$\psi_{\tau,t}(t) = \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-\tau}{s}\right), \quad s,\tau \in R, s \neq 0,$$

where s – scale factor that controls the width of the wavelet, τ – time interval. The wavelet function must satisfy the admissibility conditions [19, 41, 42].

CWT for time series $x(t)$ is defined as: $\mathrm{W}_x(\tau,s) = \frac{1}{\sqrt{|s|}}\int_{-\infty}^{+\infty} x(t)\psi^*\left(\frac{t-\tau}{s}\right)dt$.

The Wavelet Power Spectrum (WPS) provides information about the local variance of time series at each frequency. WPS describes how the time series $x(t)$ varies over the selected scale and at the selected time point. WPS is defined as the square of the absolute value of CWT:

$$\mathrm{WPS}_x(\tau,s) = \left|\mathrm{W}_x(\tau,s)\right|^2.$$

Wavelet Coherence (WC) is a powerful tool for describing the interaction between two time series and studying their co-movements in common time and frequency domains. The first step in removing the WC is the cross-wavelet transform (CRWT) calculation. CRWT of two time series $x(t)$ and $y(t)$ is defined as follows:

$$\mathrm{W}_{xy}(\tau,s) = \mathrm{W}_x(\tau,s)\mathrm{W}_y^*(\tau,s),$$

where W_x and W_y – CWT of time series $x(t)$ and $y(t)$ respectively, and the symbol * denotes complex conjugation.

In this case, the cross-wavelet power (CWP) is determined as follows:

$$\mathrm{CWP}_{xy} = \left|\mathrm{W}_{xy}(\tau,s)\right|.$$

By defining CRWT and CWP, one can enter square wavelet coherence (SWC):

$$\mathrm{R}_{xy}^2(\tau,s) = \frac{\left|S\left(s^{-1}\mathrm{W}_{xy}(\tau,s)\right)\right|^2}{S\left(s^{-1}\left|\mathrm{W}_x(\tau,s)\right|^2\right)S\left(s^{-1}\left|\mathrm{W}_y(\tau,s)\right|^2\right)},$$

where S - smoothing operator.

The wavelet coherence coefficient varies between 0 and 1, and it can be considered as the square of the local correlation coefficient between two time series. A greater value of this coefficient indicates a stronger relationship between the time series [18, 20, 43, 44].

SWC is not able to distinguish between positive and negative correlations and to determine the relationship between two time series. For this reason, the wavelet-coherence phase difference was introduced [19]:

$$\varphi_{xy}(\tau,s) = \mathrm{tang}^{-1}\left(\frac{\Im\left\{S\left(s^{-1}\mathrm{W}_{xy}(\tau,s)\right)\right\}}{\Re\left\{S\left(s^{-1}\mathrm{W}_{xy}(\tau,s)\right)\right\}}\right)$$

where $\Im$ and $\Re$ are imaginary and valid operators, respectively.

Arrows on the wavelet coherence figures represent the phase difference. Following the trigonometric convention the direction of arrows shows the relative phasing of time series and can be interpreted as indicating a lead/lag relationship: If the arrows point to the right (left), the time series are in-phase (anti-phase), i.e. they are positively or negatively correlated, respectively. If the arrows point up and right (left), this indicates that the study series are in-phase (anti-phase) and the first (second) time series leads the second (first) one. A zero phase indicates that two series move together [43].

In contrast to the two-dimensional analysis, the multiple wavelet correlation (WMC), developed by Fernandez and Macho [22], allows us to determine the general correlation that can exist at different time scales within a multivariable set of variables. WMC is defined as a single set of multivalued correlations calculated from a multivariate stochastic process $X_t = (x_{1t}, x_{2t}, \ldots, x_{nt})$. The wavelet coefficients of j level $(\mathrm{W}_{j,t})$ and scaling coefficients $(\mathrm{V}_{j,t})$ will be obtained for the maximum overlap DWT (MODWT) method. In each scale λ_j, WMC $\{\varphi_X(\lambda_j)\}$ is calculated as the square root of the regression determination coefficient in such a linear combination of wavelet coefficients $\mathrm{W}_{jt} = (w_{1jt}, w_{2jt}, \ldots, w_{njt})$ for which the determination coefficient is the maximum.

The WMC coefficient can be expressed as wavelet dispersion and covariance:

$$\varphi_X(\lambda_j) = \mathrm{Corr}\left(w_{ijt}, \hat{w}_{ijt}\right) = \frac{\mathrm{Cov}\left(w_{ijt}, \hat{w}_{ijt}\right)}{\sqrt{\mathrm{Var}\left(w_{ijt}\right)\mathrm{Var}\left(\hat{w}_{ijt}\right)}},$$

where w_{ijt} is chosen for maximum increase $\varphi_X(\lambda_j)$, and $\hat{w}_{ijt}$ denotes fitted values in the regression of w_{ijt} on the rest of the wavelet coefficients on the scale λ_j.

Similarly, allowing a lag between observed and fitted values at each scale λ_j, the WMCC is defined as follows:

$$\varphi_{X,k}\left(\lambda_j\right) = \mathrm{Corr}\left(w_{ijt}, \hat{w}_{ijt+k}\right) = \frac{\mathrm{Cov}\left(w_{ijt}, \hat{w}_{ijt+k}\right)}{\sqrt{\mathrm{Var}\left(w_{ijt}\right)\mathrm{Var}\left(\hat{w}_{ijt+k}\right)}},$$

where k is a lag between observed and fitted values of the variable selected as the criterion variable at each scale λ_j.

The consistent estimator for the wavelet multiple correlation (denoted by $\tilde{\varphi}_X\left(\lambda_j\right)$) and consistent wavelet multiple cross correlation estimator (denoted by $\tilde{\varphi}_{X,k}\left(\lambda_j\right)$) can be constructed in the same way by substituting $\varphi_X\left(\lambda_j\right)$ for $\tilde{\varphi}_X\left(\lambda_j\right)$ and $\varphi_{X,k}\left(\lambda_j\right)$ for $\tilde{\varphi}_{X,k}\left(\lambda_j\right)$ [22, 45].

The idea of recurrent neural networks (RNN) is to use sequential information. In the traditional neural network, we assume that all inputs are independent of each other. But for many tasks, this is not an optimal idea. RNN are called recursive because they perform the same task for each sequence element, with initial data dependent on previous calculations. Recurrent neural networks have a "memory" that captures information about what was calculated by this time [46-48].

The Long Short Term Memory (LSTM) networks are a special type of RNN that can study long-term dependencies. All RNN have the form of a chain of repetitive neural network modules. In a standard RNN, this repeating module has a simple structure of one layer. LSTM also has such a chain structure, but the repeating module has four layers.

The LSTM module (or cell) has 5 main components, which allows you to model both long-term and short-term data:

- the state of the cell is the internal memory of the cell, which stores both short-term memory and long-term memory;
- hidden state - this is the initial status information calculated for the current logon;
- input gateway - determines how much information from the current incoming stream enters the cell's state;
- “forget gate” - determines how much information from the current input and the previous state of the cell goes to the current state of the cell;
- output gateway - decides how much information from the current state goes into a hidden state.

The RNN can be considered as multiple copies of one network, each of which sends a message to the next one.

The back propagation (BP) neural network is an artificial intelligence algorithm widely used in prediction, in particular for advanced multiple regression analysis. It better generates complex and non-linear responses than a standard regression analysis [49]. A BP network uses the gradient method, and the learning and inertial factors are determined by experience. This affects the convergence in a BP network.

The Wavelet-based BP method uses both a wavelet-based multi-resolution analysis and multi-layer artificial neural networks. The DWT allows decomposing sequences of past data in subsequences (named coefficients) according to different frequency domains, while preserving their temporal characteristics [50].

To assess the accuracy of forecasting, two criteria are used: mean square error (RMSE), average absolute percentage error (MAPE).

$$RMSE=\sqrt{\frac{\sum_{t=1}^{N}\left(\hat{y}_t-y_t\right)^2}{N}},$$

where y_t and $\hat{y}_t$ - the actual value and the predicted value at time t, respectively, N - the size of the data set. RMSE expresses the standard deviation of the difference between predicted and actual values.

MAPE, also known as the average absolute deviation percentage (MAPD), expresses accuracy in percentages:

$$MAPE=\frac{1}{N}\sum_{t=1}^{N}\left|\frac{y_t-\hat{y}_t}{y_t}\right|.$$

MAPE measures the average absolute relative error of forecasting. RMSE and MAPE are widely used to estimate predictive accuracy. The accuracy of the model is higher when the value of RMSE and MAPE are lower [51, 52].

4 Research results

To study the relationship that causes correlations between the oil and gas market, the Dow Jones index and the US dollar index, we used Brent crude oil prices, Henry Hub gas prices, and the Dow Jones index and the US dollar index respectively. The data set consists of daily figures for the period from September 2007 to August 2019. This interval was chosen based on the fact that it covers the main fluctuations in selected markets. Fig. 1 shows the dynamics of prices and indices. We can see that, for some times, series tend to have the same trend, and in other periods, they are different. For example, from 2007 to 2008, unlike the oil and gas market, where this period was characterized by rising prices, we see a decline in the US dollar index and the relative stability of the Dow Jones index. Between 2008 and 2009 there was a sharp fall in prices on oil and gas markets and a drop in the Dow Jones index. At the same time, the US dollar index was stable for the first half of the year and then increased. In 2014-2015, the US dollar index was growing fast, the Dow Jones index was slower of it but also growing, unlike oil and gas prices that were falling. Only the Dow Jones Index from 2009 to 2018 had a pronounced rising trend, other series were more volatile.

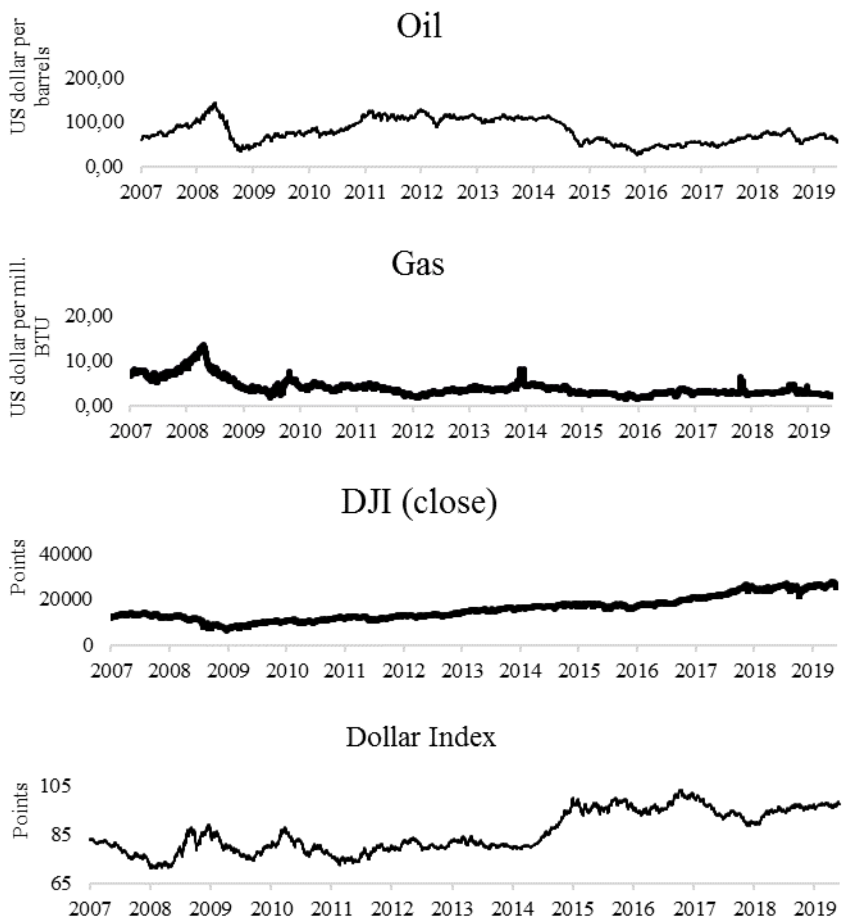

Fig. 1. Dynamics of time series

Descriptive statistics of time series logarithmic returns are given in Table 1. The average returns is positive for all series except of gas. The available average standard deviation ranges from 0.5% to 4.3%, the most volatile time series is gas price, and the US dollar index is the least volatile. Oil and gas have a positive skewness and, consequently, a long right tail. The Dow Jones Index and the US Dollar Index have a positive skewness and heavy right tail. The calculated kurtosis coefficients prove that the distributions have sharp vertices in relation to the normal distribution. In addition, the statistics of Pearson categorically rejects the null hypothesis, which assumes that the distribution of returns is normal.

Table 1. Descriptive statistics of time series returns

	Oil	Gas	Dow Jones index	US dollar index
Average value	2.28E-04	5.34E-04	3.08E-04	6.30E-05
Standard deviation	0.022	0.043	0.012	0.005
Skewness	0.347	2.590	0.066	0.022
Kurtosis	5.917	40.570	10.808	2.243
Pearson's statistics	687.930	1871.100	2313.800	5786.500

In order to study the interconnections between markets, the co-motion of the series, as well as studying whether the relationship between markets in the time/frequency domain wavelet analysis is further used. The use of wavelet analysis allows to identify patterns of evolutionary paths between selected time series, as well as to study how these variables interact at different frequencies, and how this interaction evolves over time and across different time scales. The calculations were carried out in the RStudio program environment. Morlet's mother wavelet with six levels of decomposition was used. Fig. 2 shows a wavelet power spectrum for the oil market at different time scales. Three cycles were chosen to construct the wavelet power spectrum. The first and second cycles on the middle scales are 16-32 days (monthly scale) and 32-64 days (from monthly to quarterly scale). The third cycle on a scale of 64-128 days (from a quarterly to annual scale) refers to a long-term analysis. These periods are deferred on the vertical axis of the graph, the time is indicated on the horizontal axis. The wavelet power is indicated by the color ranging from red to blue, which corresponds to regions of high and low power respectively. White contours indicate a 5% significance level. "Cone of influence", where boundary effects become important, is shown with a lighter shade. Black lines indicate power peaks. There are two distinct regions with high volatility with white circles at medium scales (16-32 days) in the end of 2008 and the beginning of 2016. The available peaks of power are due to the global crisis and the sharp drop in prices on the world market, respectively. One can also observe the high power region at the beginning of 2015 at medium scale (32-64 days). It can be explained by the long fall in oil prices when they have reached its historic minimum.

The spectrums of gas prices, the Dow Jones index and the US dollar index have regions of high power at medium scales (16-32 days, 32-64 days) in the end of 2008. Also, periods with high volatility of gas prices are observed at the same scales in 2016-2017. For the Dow Jones and US Dollar index, similar regions are in 2011, 2015, and the end of 2018.

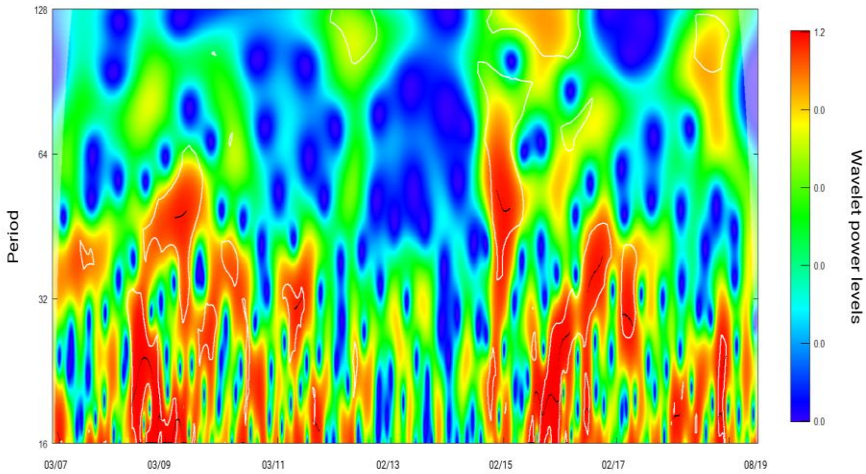

Fig. 2. Wavelet power spectrum of oil market

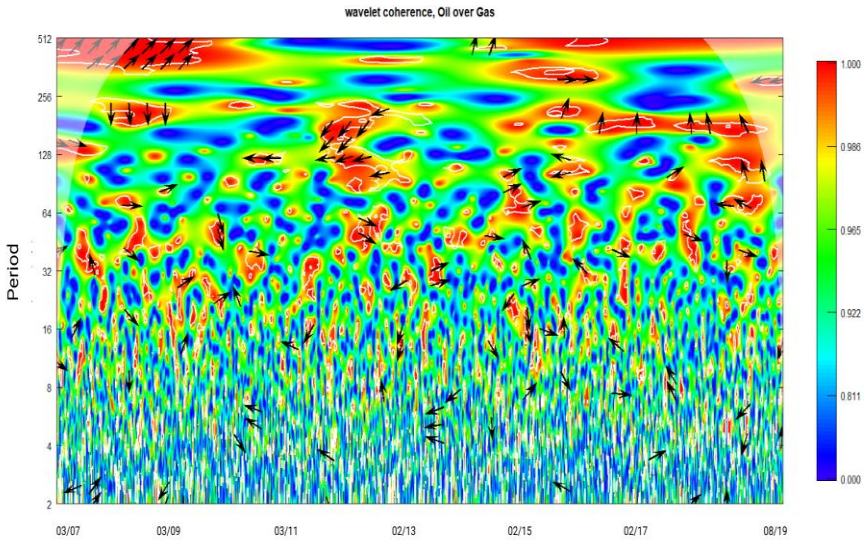

Fig. 3. Wavelet coherence between returns of oil and gas prices

The next stage of the study is the calculation of wavelet coherence for the logarithmic returns of time series. Graphs of spectra are constructed in the same way: time and period are marked on the axes. In this case, more periods were included, namely: 2-4 days (intraweek scale), 4-8 days (weekly scale), 8-16 days (two-week scale), 16-32 days (monthly scale), 32-64 days (from monthly to quarterly scale), 64-128 days (from quarterly to two-quarter scale), 128-256 days (from two to three quarterly scale) and 256-512 days (annual scale). The arrows indicate the phase difference between the two time series.

Figs. 3-5 shows the degree of similarity and phase relationships between the logarithmic returns of oil and gas prices, oil prices and the US dollar index, oil prices, and the Dow Jones index respectively.

The coherence between the returns of oil and gas prices (Fig. 3) is strong at high scales (128-256 days, 256-512 days). Several "islands" of high coherence can be identified at medium scales in 2008, 2012 and 2015-2017. At the same time, in most cases, the direction of the arrows indicates that changes in oil prices lead to changes in the gas market, that is, the oil prices are leading.

Fig. 4 shows the wavelet coherence between the returns of oil and the US dollar index. One can see the similar picture, but in this case, the series are in the antiphase. That is, the volatility of the US dollar index causes changes in the oil market. At low scales, the correlation is weak, strong correlation periods are observed in 2008 and over the period 2015-2017 at medium and high scales.

Analyzing the coherence between the returns of the oil prices and the Dow Jones index (Fig. 5), we can say that fluctuations in oil prices affect the volatility of the Dow Jones index, that is, the series correlate positively. Three high-coherence periods can be distinguished: 2008, mid-2011, 2016 and mid-2018-2019 at medium and high scales. At low scales, the correlation is small.

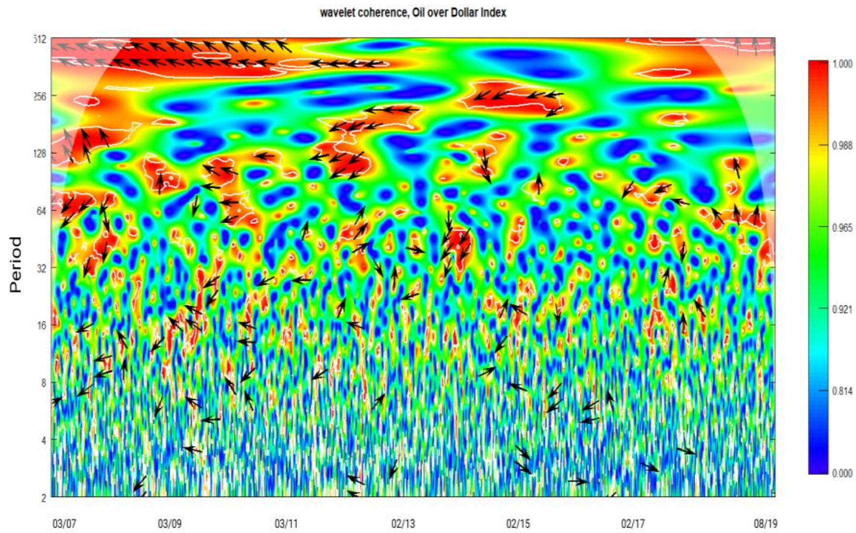

Fig. 4. Wavelet coherence between returns of oil price and the US dollar index

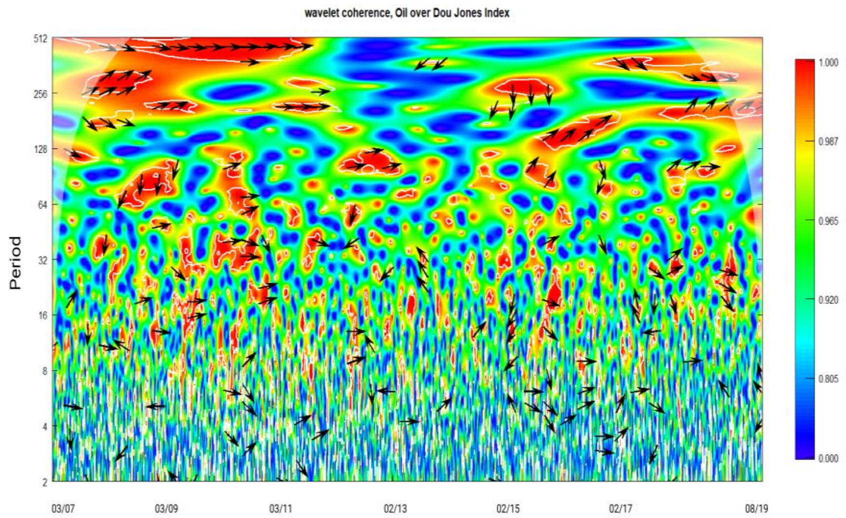

Fig. 5. Wavelet coherence between returns of oil price and the Dow Jones index

Interaction of the time series of gas prices and the US dollar index is weak at low and medium scales, but significant at high one. There is a period of high coherence in the period 2008-2009. In this case, the arrows are mainly directed upwards and to the left. It indicates the two series are in antiphase. The US dollar index is a leading series, its volatility affects the gas market.

The correlation between returns of gas prices and the Dow Jones index is similar: it is negligible or absent at all low scales, but strong at medium and high. There is a marked area of high coherence at high scales in 2008. It is interesting, the gas market is leading at medium scales, and the Dow Jones index is leading at high ones.

At medium and high scales, the returns of the US dollar index and the Dow Jones index are both in antiphase (the arrows are mostly directed to the left). It means that the second series is the lead. There is a pronounced period of high coherence at medium and high scales in 2008. At low scales, the correlation is small or absent.

So, comparing the obtained results, we can say that high coherence is observed in both crisis and non-crisis periods. The highest coherence of the series returns is marked at medium and high scales during 2008. In most cases, at these scales, oil prices and the US dollar index, gas prices and the US dollar index, as well as the US dollar and Dow Jones indexes, move in the antiphase. However, there are periods with a bidirectional relationship between the series at the medium and high scales. At the same time, the oil market leads the gas market. The US dollar index influences (is leading) the formation of oil and gas prices. In turn, oil prices affect the value of the Dow Jones index.

In [53], wavelet coherence was studied for a smaller range of data. Increasing the length of the series over a 6-month period is not significant changed the overall results. The updated coherence spectra were confirmed trends of co-movement of the studied series at medium and high scales.

The wavelet multiple correlation was obtained for the diferent groups of time series. Fig. 6 presents the wavelet multiple correlation for all four markets together. On a horizontal axis, the 8 decomposition levels by the Daubechies(4) wavelet are plotted. On the vertical axis, the wavelet multiple correlation coefficient is marked. The blue lines show the upper and lower limits of the 95% confidence interval. The black line connects the value of the multiple correlation between the given time series at a certain scale. Below there is indicated what market is leading for a certain period. At medium scales (32-64 days, 64-128 days) the US dollar index is ahead, at high scales (128-256 days, 256-512 days) the oil market is leading. At high scales, co-movement is almost linear; the multiple correlation reaches a value of about 0.9. We can conclude that the combination of financial (gas and oil market) and stock markets (the Dow Jones Index and the US dollar index) makes them more integrated.

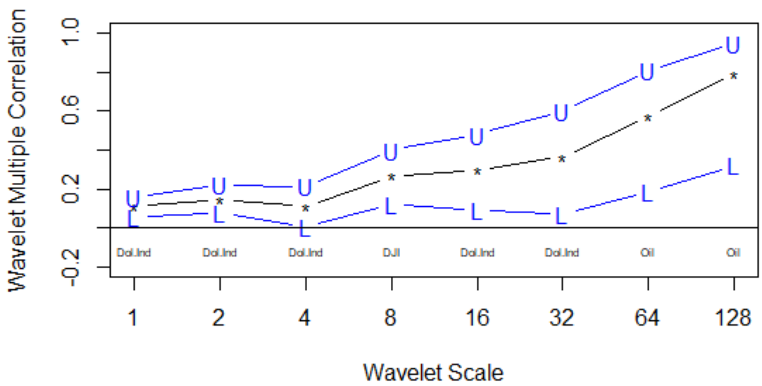

Fig. 6. Wavelet multiple correlation for all time series returns

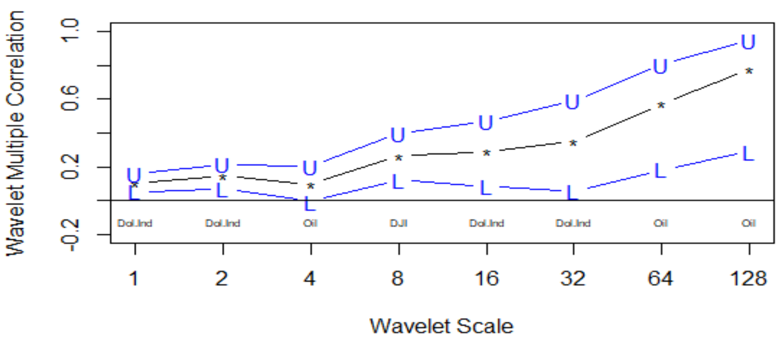

Fig. 7. Wavelet multiple correlation for oil, DI, DJI returns

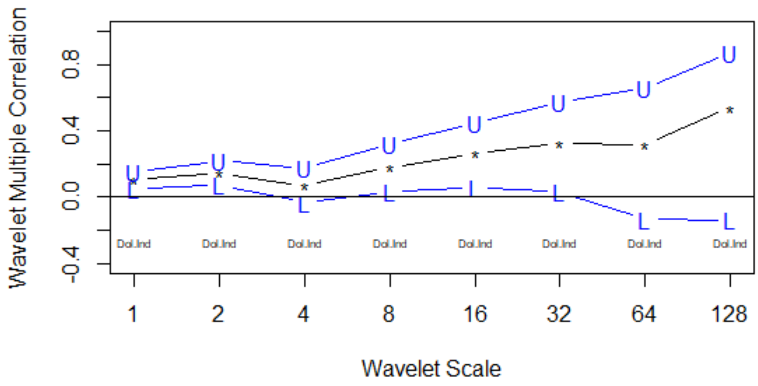

Fig.8. Wavelet multiple correlation for gas, DI, DJI returns

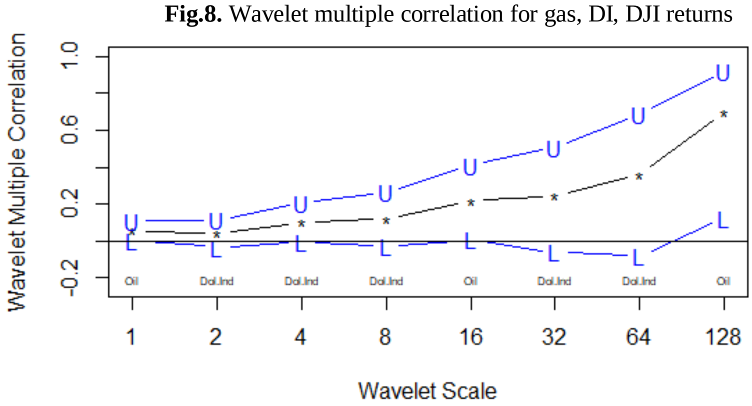

Fig.9. Wavelet multiple correlation for gas, oil, DI returns

The multiple wavelet correlation of the oil market, the US dollar index and the Dow Jones index (Fig.7) are small at low scales (2-4 days, 4-8 days, 8-16 days) and medium scales (16-32, 32-64 days) with a value of about 0.2 and at medium scales (64 -128 days) with a value of about 0.3. In this case, the multiple correlation values increase at high scales (128-256 days, 256-512 days), starting from the value of 0.4 and reaching a maximum value of 0.8. The leading market is the oil one. Consequently, at high scales, the existence of a linear relationship between markets cannot be ruled out. Such a result can be interpreted as the integration between these markets in the sense that the returns on one market can be fully determined by the overall efficiency in other markets for periods greater than a year.

The wavelet multiple correlation of the gas market, the US dollar index and the Dow Jones index at low and medium scales is small, only on a high scale it reaches a maximum of 0.5 (Fig.8). The leading is the US dollar index. The multiple correlations of the gas, oil and the Dow Jones index, as well as the multiple correlation of the gas, oil and the US dollar index, share common features (Fig.9). Namely, there is a small correlation at low and medium scales and a gradual increase of a correlation at high scales. In the first case, the oil market is steadily leading. In the second case, the US dollar index and oil price are leaders at scales (128-256 days) and (256-512 days) respectively.

The wavelet multiple cross-correlations for all time series returns at different levels of wavelet decomposition with lags up to one month are shown in Fig. 10. In the upper left corner of each graph a variable that maximizes the multiple correlation with the linear combination of the remaining variables is represented. Thus, it is identifyted a potential leader or follower for the entire system. The red lines correspond to the upper and lower limits of the 95% confidence interval. At levels 1-3, the oil market maximizes multiple correlations against a linear combination of other markets at all levels of the wavelet decomposition. At levels 4-5, the Dow Jones index has the potential to lead or lag the other markets, at level 6 the maximizing variable is the US dollar index. All variables are positively correlated on all scales, and they tend to co-movement. It is also noticeable that the correlation weakens with increasing lag. There is a small skewness (left-sided) at levels 5-6, indicating that the Dow Jones Index and the US dollar index are lagging behind the oil and gas market. Accordingly, oil prices can be viewed as a leading barometer of global mood; changes in this market affect the volatility of gas prices, the US dollar and the Dow Jones indexes.

At the next stage, a comparison of the predictive capabilities of various neural networks is made. The Long Short Term Memory (LSTM) and Wavelet Based Back Propagation (WBP) neural networks are considered. Brent oil prices and the US dollar index, as leaders of co-movement, daily from March 1, 2007 to August 16, 2019 are used. The LSTM neural network was modeled in the RStudio software environment with Keras and TensorFlow packages.

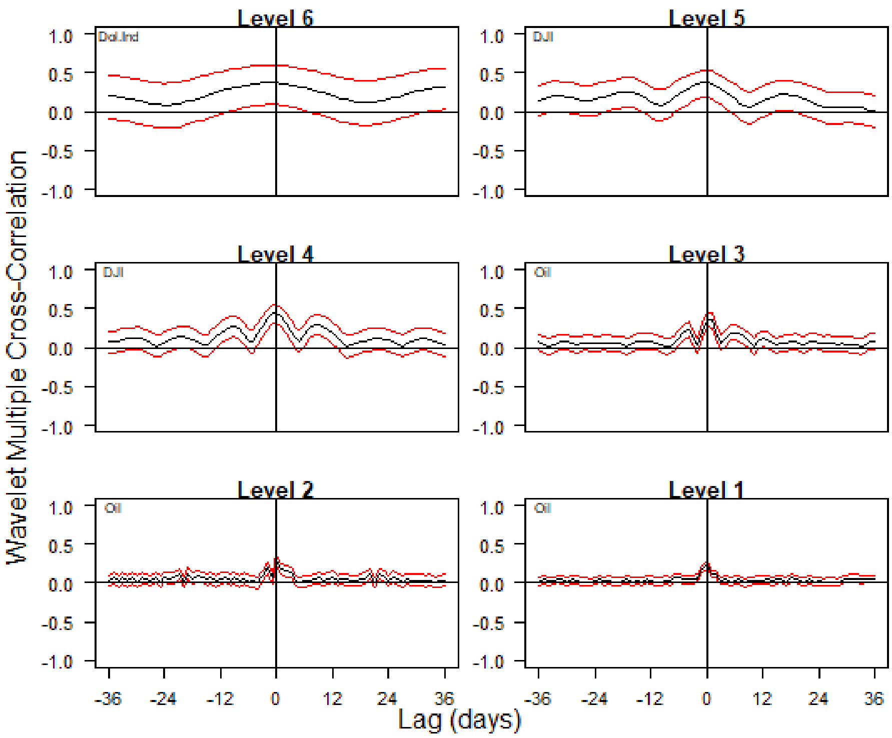

Fig. 10. Wavelet multiple cross-correlation for all time series returns

Before the beginning of the simulation process, it is necessary to prepare the input data. First of all, it is necessary to convert data to the stationary ones by finding the difference of the first order. The next step is to create an additional first-order lag variable, since LSTM involves learning a neural network with a teacher. All time series are divided into training and test parts. It was decided that 90% of the data was used to train the network, and, accordingly, 10% - for testing.

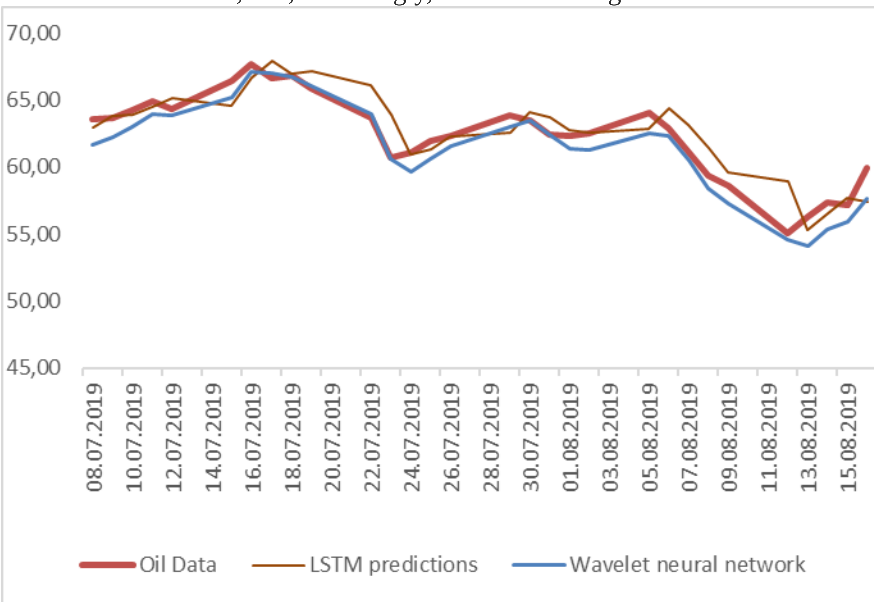

Fig. 11. Real data and forecasts of oil prices by WBP and LSTM methods

Pre-processing data also includes operations of normalization and data recovery. The network architecture consists of an input layer, one hidden layer, and an output layer. The hidden layer contains memory cells and corresponding device blocks that are characteristic of the recurrent neural network.

The WBP modeling was performed in the Alyuda NeuroIntelligence environment. The neural network architecture consisted of an input layer, one hidden layer, and an output layer. Fig. 11-13 show the 30-day forecasting result for oil prices, the US dollar index and the Dow Jones index test data. Table 2 presents the RMSE and MAPE errors which were calculated for the series and for considered forecasting methods.

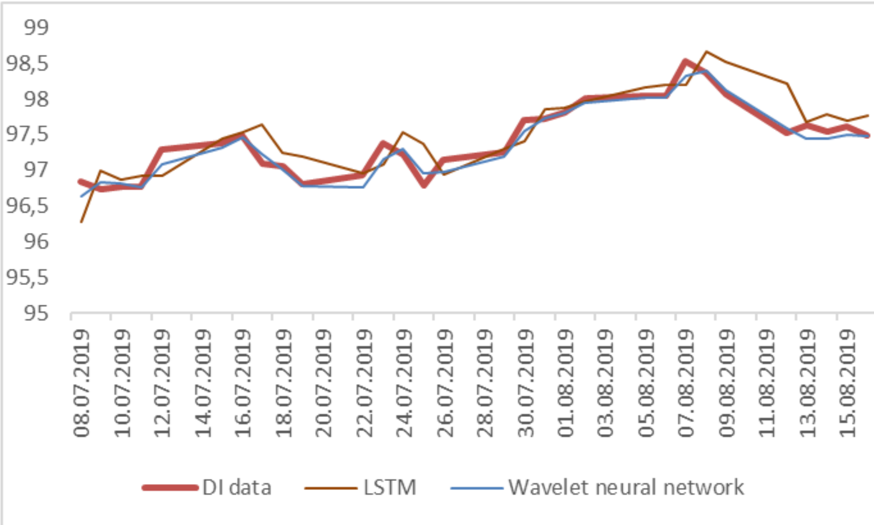

Fig. 12. Real data and forecasts of the US dollar index by WBP and LSTM methods

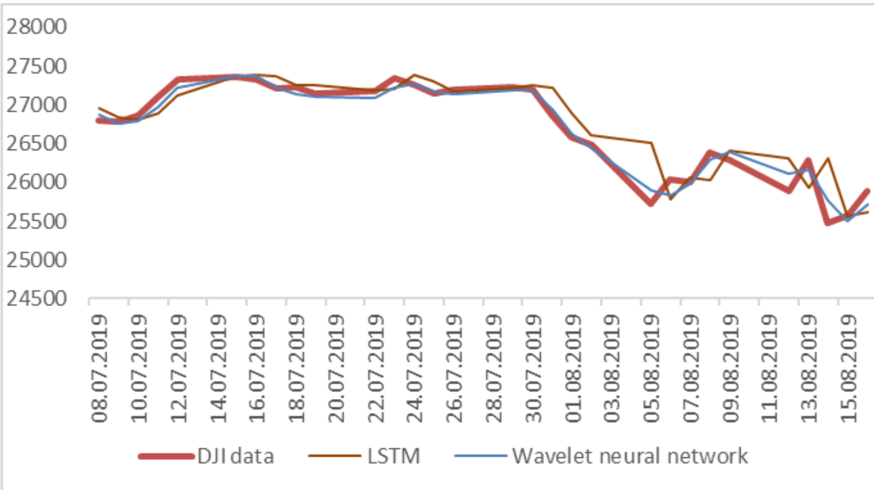

Fig. 13. Real data and forecasts of the Dow Jones index by WBP and LSTM methods

Table 2. Forecasting errors

	LSTM		WBP	
	RMSE	MAPE	RMSE	MAPE
Brent	1.48	0.0186	1.10	0.0153
DJI	280.06	0.0073	108.85	0.0032
DI	0.30	0.0025	0.12	0.0010

In general, empirical analysis shows that “deep learning” neural network gives possibility to build qualitative models with high forecasting accuracy. Due to the fact that with each iteration new nonlinear interconnections are constructed, we can achieve rather small values of errors. However, the comparison of forecasting errors suggests that the WBP method on short horizons gives better results.

Compared with the results of forecasting in [53] for a similar time period, a slight decrease in errors can be observed. However, the accuracy of the oil price forecasting is somewhat lower for both methods. This result is expected because the oil market depends on geopolitical factors that are difficult to account for time series forecasting.

5 Conclusion

The use of wavelet techniques for studing the dynamics of the time series allowed to establish some correlation relationships between volatility in the relevant markets. By means of discrete wavelet transform and continuous wavelet transform, the wavelet power spectrum of each series was constructed, wavelet coherence for time series paires was investigated, and wavelet multiple correlation was determined.

Four time series were selected for the study: Brent crude oil prices, Henry Hub gas prices, the Dow Jones Index and the US Dollar Index. Their dynamics over time characterize the behavior of the respective markets.

In general, four global markets show a similar picture in terms of the wavelet power spectrum, which is confirmed by the high level of volatility at the medium scales. The levels of high volatility at the same intervals explain that there is a link between the changes in these markets, and the global economy is vulnerable to oil and gas prices, the value of the dollar index and the Dow Jones index.

High coherence of the series is observed both in crisis and in non-crisis periods. The largest correlation is marked at medium and hight scales during 2008. With the interaction of oil and gas markets, the oil market is leading. The US dollar index influences (is leading) the formation of oil and gas prices. In turn, oil and gas prices affect the value of the Dow Jones index. There are periods with a bidirectional relationship between the oil and gas markets, the Dow Jones index and the US dollar index at the medium and high scales.

Wavelet multiple correlations between the four markets are positive at all scales and become stronger with increasing horizons of time. The oil market is a potential leader in the medium and long term. Therefore, we can say that changes in this market affect the volatility of other markets. At high scales, the co-movement of the series is almost linear. The combination of financial markets (gas and oil market) and stock markets (the Dow Jones Index and the US dollar index) makes them increasingly integrated.

The wavelet multiple cross-correlations for all time series returns at different levels of wavelet decomposition with leads and lags up to one month were computed. There is a small left-sided skewness in high periods, indicating that the Dow Jones Index and the dollar index are lagging behind the oil and gas market. According to the research results, oil prices can be considered as a leading barometer of world sentiment, changes in this market affect the volatility of gas prices, the US dollar and the Dow Jones indexes.

For series-leaders, forecasting models based on neural network of deep learning and Wavelet based Back Propagation were built. Comparison of the forcasting errors suggests that the application of both methods on short horizons gives good modeling results.

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